Written Exam Economics Summer 2019

Labour Economics

June 1, 2019 (10 am-10 pm)

This exam question consists of 4 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
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- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

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Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

1 Labor supply and taxation

Consider a worker, who gets utility from consumption and leisure. Let c denote consumption, l denote leisure, and let the total hours available be given by l_0. The hours worked is $h = l_0 - l$ and the worker can freely decide how many hours to work at the wage w. The worker has the utility function

$$U(c, l) = c - \frac{\gamma}{\eta}(l_0 - l)^\eta$$

where $\gamma > 0$ and $\eta > 1$ are parameters. The price of the consumption good is normalized to 1, such that the real wage is given by w. The worker's budget constraint is

$$c \leq w(l_0 - l)$$

1. What is the worker's optimal choice of hours of work and consumption?

2. Calculate the Marshallian wage elasticity of hours.

3. Consider how the choice of hours is affected if the worker has to pay the tax rate τ, such that the after-tax wage rate is given by $(1 - \tau) w$. How is the Marshallian elasticity related to the impact of taxation on the hours of work?

4. Now suppose that pre-tax earnings $z = wh$ below z^* are not taxed, but that only pre-tax earnings above z^* are taxed by the tax rate τ. Let the optimal hours without taxes (as in question 1) be denoted h_1 and denote the optimal choice of hours when all income is taxed by τ (as in question 3) by h_2. What are the worker's optimal choice of hours work for each of the following three cases: a) $wh_1 < z^*$, b) $wh_2 > z^*$, and c) $wh_1 \geq z^*$ and $wh_2 \leq z^*$.

5. We assume that all workers receive the same wage and have the same η, but that the parameter γ varies between individuals. What is the interval of γ satisfying that $wh_1 \geq z^*$ and $wh_2 \leq z^*$. Is the interval width increasing or decreasing in τ? Give an interpretation of this result.

2 The matching model with search intensity

An unemployed worker can increase the chances of finding a job by searching more intensively. Assume that the probability of finding a job is given by $\lambda(s_i)$, where s_i is worker i's search effort. However, searching harder carries a cost for the unemployed. We assume that search costs are increasing and convex in the search effort, $c'(s_i) > 0$ and $c''(s_i) > 0$. It is assumed that matching is governed by the matching function $M(sU, V)$, where s is the average (or common) search intensity, U is the number of unemployed, and V is the number of vacancies. We assume that the matching function is increasing in each of its inputs and that it exhibits constant returns to scale. We also assume that $M(0, V) = M(sU, 0) = 0$. The equilibrium unemployment rate is given by

$$u = \frac{q}{q + \lambda(s)}$$

where q is the exogenous job destruction rate.

The value of being unemployed for person i with search effort s_i, $V_u(s_i)$, is given by

$$rV_u(s_i) = z - c(s_i) + \lambda(s_i)[V_e - V_u(s_i)]$$

where r denotes the discount rate, z is the flow income as unemployed, and V_e is the value of being employed. The latter is given by the following Bellman equation

$$rV_e = w + q[V_u(s_i) - V_e]$$

where w denotes the wage and q is the exogenous job destruction rate.

For the firms, the value of a vacancy is denoted Π_v, while the value of a filled job is Π_e. We have

$$r\Pi_v = -h + m\left(\frac{\theta}{s}\right)(\Pi_e - \Pi_v)$$

where h is the flow cost of a vacancy and $m\left(\frac{\theta}{s}\right) \equiv \frac{M(sU,V)}{V}$ and where $\theta \equiv \frac{V}{U}$. The Bellman equation for a filled job is given by

$$r\Pi_e = y - w + q(\Pi_v - \Pi_e)$$

where y is the flow productivity and w the flow wage. We assume free entry in vacancy creation.

1. Use the matching function to show that the job arrival rate for worker i with search intensity s_i can be written as $\lambda(s_i) \equiv \lambda(s_i; \theta, s) = \frac{s_i}{s}\theta m\left(\frac{\theta}{s}\right)$.[1]

2. Derive the first-order condition for the worker's optimal search intensity. Interpret your result.

3. We are only interested in a symmetric equilibrium, where all unemployed workers are searching with the same intensity $s_i = s$. Hence, as you continue, you should evaluate the first-order condition for s_i in $s_i = s$. We would like to understand how the search intensity is affected by the labor market tightness. Denote the match surplus by S. Then, use that $V_e - V_u = \gamma S = \frac{\gamma}{1-\gamma}(\Pi_e - \Pi_v)$ to show that the first-order condition can be written as

$$sc'(s) = \frac{\gamma}{1-\gamma}\theta h \tag{1}$$

   Interpret the relationship between the search intensity and the labor market tightness.

[1] Notice that above we have used the short-hand notation $\lambda(s_i)$ rather than $\lambda(s_i; \theta, s)$ to simplify the exposition. The job arrival rate for worker i is a function of s_i, θ, and s.

4. Derive the firms' vacancy supply curve. Provide an interpretation of this equation.

Wages are determined by Nash bargaining over the match surplus when the worker and firm meet. The worker is assumed to have the bargaining power γ. It can be shown that the wage is given by

$$w = [z - c(s)] + (y - [z - c(s)]) \frac{\gamma \left[r + q + \theta m \left(\frac{\theta}{s}\right)\right]}{r + q + \gamma \theta m \left(\frac{\theta}{s}\right)}$$

5. Combine the vacancy supply curve and the wage equation to eliminate the wage. The resulting equation together with equation (1) determine the search intensity and the labor market tightness. Depict these two equations in a diagram in (s, θ) space. Discuss whether we have a unique solutions for s and θ.

6. What is the effect of a higher productivity on the search intensity, the labor market tightness, the wage and the unemployment rate? Use graphs to illustrate the equilibrium effects of a higher productivity.